

Twist-3 distribution amplitudes of K^* and ϕ mesons

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ABSTRACT: We present a systematic study of twist-3 light-cone distribution amplitudes of K^* and ϕ mesons in QCD. The structure of SU(3)-breaking corrections is studied in detail. Non-perturbative input parameters are estimated from QCD sum rules. As a by-product, we update the parameters describing the twist-3 distribution amplitudes of the ρ meson. We also review and update predictions for the twist-2 distribution amplitudes of ρ , K^* and ϕ .

KEYWORDS: QCD, Weak Decays, Kaon Physics, Conformal and W Symmetry.

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1. Introduction

In recent years SU(3)-symmetry-breaking in processes involving light vector mesons has attracted increasing interest. For some of these processes, for instance the heavy-meson decays $B \rightarrow \rho\gamma$ vs. $B \rightarrow K^*\gamma$, the uncertainty in SU(3) breaking is presently the dominant source of theoretical error [1, 2]. These and related decays, like $B \rightarrow (\rho, K^*)\ell^+\ell^-$, are dominated, at short distance, by flavour-changing neutral-current transitions which are heavily suppressed in the Standard Model (SM) (they occur only at loop-level) and hence are very sensitive to potential effects from new physics. All these decays will be studied in detail at the LHC, with the aim, in the case of the discovery of new physics in the TeV range, to elucidate its flavour structure, or, in the case of continued absence of new particles in direct searches, to constrain their possible masses and couplings. In any case, good theoretical control over the SM predictions of such decays is vital. The current theoretical approaches to describe them all rely, in one way or the other, on their interpretation as hard exclusive reactions, with the hard scale set by the heavy quark or meson mass which leads to an expansion in terms of the inverse hard scale.¹

¹This also applies to lattice calculations, see ref. [3].

In a perturbative framework, the method of choice for calculating matrix elements of $B \rightarrow$ light meson transitions is QCD factorisation, which enters QCD sum rules on the light cone [4], QCD factorisation for non-leptonic and radiative B decays [5] and perturbative QCD factorisation [6]. One important ingredient in these calculations are light-cone hadron distribution amplitudes (DAs) which describe the momentum-fraction distribution of partons at zero transverse separation in a particular Fock state, with a fixed number of constituents. DAs are ordered by increasing twist; the leading-twist-2 meson DA $\phi_{2;M}$, which describes the momentum distribution of the valence quarks in the meson M , is related to the meson's Bethe-Salpeter wave function $\phi_{M,BS}$ by an integral over transverse momenta:

$$\phi_{2;M}(u, \mu) = Z_2(\mu) \int^{|k_\perp| < \mu} d^2 k_\perp \phi_{M,BS}(u, k_\perp).$$

Here u is the quark momentum fraction, Z_2 is the renormalisation factor (in the light-cone gauge) for the quark-field operators in the wave function, and μ denotes the renormalisation scale. For pseudoscalar mesons, one has one twist-2 DA, whereas for vector mesons there are two, $\phi_{2;M}^\perp$ and $\phi_{2;M}^\parallel$, one for each independent polarisation state of the vector meson, transverse and longitudinal, respectively. In this paper we study the twist-3 distribution amplitudes of the vector mesons ρ , K^* and ϕ , with a particular emphasis on SU(3) (and G-parity) breaking effects, and also update earlier results on twist-2 parameters. We do not differentiate between ρ and ω mesons as their DAs only differ by the numerical values of hadronic parameters which, using the currently available theoretical methods, coincide within errors. Our paper is an extension of ref. [7] to vector-meson DAs and finalises the preliminary results for twist-3 parameters quoted earlier in refs. [1, 2, 8]. The results for twist-4 DAs will be published elsewhere.

The study of vector-meson DAs has attracted less attention than that of pseudoscalar DAs. The leading twist-2 DAs of ρ have been investigated in ref. [9], correcting a mistake in the earlier literature [10]. The structure of twist-3 DAs of ρ , K^* and ϕ and their relation to twist-2 DAs have been studied in ref. [11], but did not include all SU(3)-breaking effects. In this paper, we complete the analysis of ref. [11] by including all G-parity and SU(3) breaking corrections for K^* and ϕ and also providing numerical values of all hadronic parameters.

Our paper is organised as follows: in section 2 we introduce notations and review the status of the twist-2 parameters. In section 3 we provide parametrisations of all twist-3 DAs to NLO in chiral expansion and including all G-parity breaking contributions. In section 4 we discuss numerical models for these DAs, based on the results from QCD sum rules. We conclude and summarise in section 5. The appendices contain a discussion of the renormalisation-scale dependence of the twist-3 parameters, which is also affected by SU(3)-breaking corrections, and the QCD sum rules used to derive numerical values for all parameters.

2. General framework and twist-2 DAs

In this section we introduce notations and review the status of twist-2 parameters.

2.1 Kinematics and notations

Light-cone meson DAs are defined in terms of matrix elements of non-local light-ray operators extended along a certain light-like direction z_μ , $z^2 = 0$, and sandwiched between the vacuum and the meson state. We adopt the generic notation

$$\phi_{t;M}^\lambda(u), \psi_{t;M}^\lambda(u), \dots \quad (2.1)$$

and

$$\Phi_{t;M}^\lambda(\underline{\alpha}), \Psi_{t;M}^\lambda(\underline{\alpha}), \dots \quad (2.2)$$

for two-particle and three-particle DAs, respectively. The superscript λ denotes the polarisation of the vector meson: $\lambda = \parallel$ (\perp) for longitudinal (transverse) polarisation. The first subscript $t = 2, 3, 4$ stands for the twist; the second one, $M = \rho, K^*, \dots$, specifies the meson. For definiteness, we will write most expressions for K^* mesons, i.e. $s\bar{q}$ bound states with $q = u, d$. Whenever relevant, we will include quark mass corrections in the form $m_s \pm m_q$, which allows one to obtain the results for ϕ by $m_q \rightarrow m_s$. We do not include the ω , as all formulas for DAs coincide with those of the ρ ; the difference is in the numerical values of the hadronic parameters which, at least in the framework of QCD sum rules, coincide with those for the ρ except for small differences due to the difference in meson masses. The variable u in the definition of two-particle DAs always refers to the momentum fraction carried by the quark, $u = u_s$, whereas $\bar{u} \equiv 1 - u = u_{\bar{q}}$ is the antiquark momentum fraction. The set of variables in the three-particle DAs, $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\} = \{\alpha_s, \alpha_{\bar{q}}, \alpha_g\}$, corresponds to the momentum fractions carried by the quark, antiquark and gluon, respectively.

To facilitate the light-cone expansion, it is convenient to use light-like vectors p_μ and z_μ instead of the meson's 4-momentum P_μ and the coordinate x_μ :

$$\begin{aligned} z_\mu &= x_\mu - P_\mu \frac{1}{m_{K^*}^2} \left[xP - \sqrt{(xP)^2 - x^2 m_{K^*}^2} \right] = x_\mu \left[1 - \frac{x^2 m_{K^*}^2}{4(zp)^2} \right] - \frac{1}{2} p_\mu \frac{x^2}{zp} + \mathcal{O}(x^4), \\ p_\mu &= P_\mu - \frac{1}{2} z_\mu \frac{m_{K^*}^2}{pz}. \end{aligned} \quad (2.3)$$

The meson's polarization vector $e^{(\lambda)}$ can be decomposed into projections onto the two light-like vectors and the orthogonal plane as follows:

$$e_\mu^{(\lambda)} = \frac{e^{(\lambda)} z}{pz} p_\mu + \frac{e^{(\lambda)} p}{pz} z_\mu + e_{\perp\mu}^{(\lambda)} = \frac{e^{(\lambda)} z}{pz} \left(p_\mu - \frac{m_{K^*}^2}{2pz} z_\mu \right) + e_{\perp\mu}^{(\lambda)}. \quad (2.4)$$

We also need the projector $g_{\mu\nu}^\perp$ onto the directions orthogonal to p and z ,

$$g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{1}{pz} (p_\mu z_\nu + p_\nu z_\mu), \quad (2.5)$$

and will often use the notations

$$a_z \equiv a_\mu z^\mu, \quad b_p \equiv b_\mu p^\mu \quad (2.6)$$

for arbitrary Lorentz vectors a_μ and b_μ .

The dual gluon field strength tensor is defined as $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$. Our convention for the covariant derivative is $D_\mu = \partial_\mu - igA_\mu$. Sometimes, a different convention for the sign of g is used in the literature, with $D_\mu = \partial_\mu + igA_\mu$. The sign of g is relevant for all three-particle twist-3 DAs, see table 1.

2.2 Conformal expansion and the structure of SU(3)-breaking corrections

A convenient tool to study DAs is provided by conformal expansion, see ref. [12] for a review.² The underlying idea is similar to partial-wave decomposition in quantum mechanics and allows one to separate transverse and longitudinal variables in the Bethe-Salpeter wavefunction. The dependence on transverse coordinates is formulated as scale dependence of the relevant operators and is governed by renormalisation-group equations, the dependence on the longitudinal momentum fractions is described in terms of irreducible representations of the corresponding symmetry group, the collinear conformal group $SL(2, \mathbb{R})$.

To construct the conformal expansion for an arbitrary multi-particle distribution, one first has to decompose each constituent field into components with fixed Lorentz-spin projection onto the light-cone. Each such component has conformal spin

$$j = \frac{1}{2}(l + s),$$

where l is the canonical dimension and s the (Lorentz-) spin projection. In particular, $l = 3/2$ for quarks and $l = 2$ for gluons. A quark field is decomposed as $\psi_+ \equiv \Lambda_+\psi$ and $\psi_- = \Lambda_-\psi$ with spin projection operators $\Lambda_+ = \gamma_p\gamma_z/(2pz)$ and $\Lambda_- = \gamma_z\gamma_p/(2pz)$, corresponding to $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $G_{z\perp}$ corresponds to $s = +1$, $G_{p\perp}$ to $s = -1$, and both $G_{\perp\perp}$ and G_{zp} correspond to $s = 0$. Multi-particle states built of fields with definite Lorentz-spin projection can be expanded in irreducible representations of $SL(2, \mathbb{R})$ with increasing conformal spin. The explicit expression for the DA of an m -particle state with the lowest possible conformal spin $j = j_1 + \dots + j_m$, the so-called asymptotic DA, is given by [14]

$$\phi_{\text{as}}(\alpha_1, \alpha_2, \dots, \alpha_m) = \frac{\Gamma(2j_1 + \dots + 2j_m)}{\Gamma(2j_1) \dots \Gamma(2j_m)} \alpha_1^{2j_1-1} \alpha_2^{2j_2-1} \dots \alpha_m^{2j_m-1}. \quad (2.7)$$

Multi-particle irreducible representations with higher spin $j + n, n = 1, 2, \dots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$), which are orthogonal over the weight function (2.7). For the twist-2 and 3 two-particle DAs these are Gegenbauer polynomials, whereas the twist-3 three-particle DAs get expanded in Appell polynomials.

In this paper we are particularly interested in SU(3)-breaking corrections to DAs. These corrections come from different sources:

- SU(3) breaking of hadronic parameters: these effects are partially known for twist-2 parameters, see refs. [10, 11, 15], but have not been studied for twist-3 parameters before;

²See ref. [13] for an alternative approach not based on conformal expansion.

- G-parity breaking parameters: these are of parametric order $m_s - m_q$ and vanish in the limit of equal quark mass, i.e. for ρ and ϕ . For twist-2 DAs, these have been calculated, to lowest order in the conformal expansion in refs. [10, 11, 15–17]; they are unknown for twist-3 DAs;³
- explicit quark mass corrections in $m_s \pm m_q$ to DAs and evolution equations: these affect only higher-twist DAs and are induced by the QCD equations of motion (EOM) which relate twist-3 DAs to each other and to twist-2 DAs, see section 3. The mass corrections to vector meson DAs have been calculated to twist-3 accuracy in ref. [11]; the effect on the evolution of DAs under a change of the renormalisation scale so far has only been investigated for pseudoscalar DAs [7].

We shall study all these effects in this paper.

Let us now see how these corrections affect twist-2 DAs.

2.3 Twist-2 distributions

The twist-2 DAs $\phi_{2;K^*}^{\parallel,\perp}$ of K^* mesons are defined in terms of the following matrix elements of non-local operators ($\xi = 2u - 1$) [11]:

$$\begin{aligned}
 \langle 0 | \bar{q}(x) \gamma_\mu s(-x) | K^*(P, \lambda) \rangle = & \\
 & f_{K^*}^{\parallel} m_{K^*} \left\{ \frac{e^{(\lambda)} x}{Px} P_\mu \int_0^1 du e^{i\xi Px} \left[\phi_{2;K^*}^{\parallel}(u) + \frac{1}{4} m_{K^*}^2 x^2 \phi_{4;K^*}^{\parallel}(u) \right] \right. \\
 & + \left(e_\mu^{(\lambda)} - P_\mu \frac{e^{(\lambda)} x}{Px} \right) \int_0^1 du e^{i\xi Px} \phi_{3;K^*}^{\perp}(u) \\
 & \left. - \frac{1}{2} x_\mu \frac{e^{(\lambda)} x}{(Px)^2} m_{K^*}^2 \int_0^1 du e^{i\xi Px} \left[\psi_{4;K^*}^{\parallel}(u) + \phi_{2;K^*}^{\parallel}(u) - 2\phi_{3;K^*}^{\perp}(u) \right] \right\}, \tag{2.8}
 \end{aligned}$$

$$\begin{aligned}
 \langle 0 | \bar{q}(x) \sigma_{\mu\nu} s(-x) | K^*(P, \lambda) \rangle = & \\
 & i f_{K^*}^{\perp} \left\{ (e_\mu^{(\lambda)} P_\nu - e_\nu^{(\lambda)} P_\mu) \int_0^1 du e^{i\xi Px} \left[\phi_{2;K^*}^{\perp}(u) + \frac{1}{4} m_{K^*}^2 x^2 \phi_{4;K^*}^{\perp}(u) \right] \right. \\
 & + (P_\mu x_\nu - P_\nu x_\mu) \frac{e^{(\lambda)} x}{(Px)^2} m_{K^*}^2 \int_0^1 du e^{i\xi Px} \left[\phi_{3;K^*}^{\parallel}(u) - \frac{1}{2} \phi_{2;K^*}^{\perp}(u) - \frac{1}{2} \psi_{4;K^*}^{\perp}(u) \right] \\
 & \left. + \frac{1}{2} (e_\mu^{(\lambda)} x_\nu - e_\nu^{(\lambda)} x_\mu) \frac{m_{K^*}^2}{Px} \int_0^1 du e^{i\xi Px} \left[\psi_{4;K^*}^{\perp}(u) - \phi_{2;K^*}^{\perp}(u) \right] \right\}. \tag{2.9}
 \end{aligned}$$

All other DAs in the above relations are of twist 3 or 4. We have neglected all terms in the light-cone expansion which are of twist 5 or higher. The normalisation of all DAs is given by

$$\int_0^1 du \phi(u) = 1. \tag{2.10}$$

³The results given in refs. [1, 2, 8] were preliminary versions of those obtained in this paper.

The above DAs are related to those defined in refs. [11, 18] by

$$\begin{aligned} \phi_{2;K^*}^{\parallel(\perp)} &= \phi_{\parallel(\perp)}, & \phi_{3;K^*}^{\parallel} &= h_{\parallel}^{(t)}, & \psi_{4;K^*}^{\parallel} &= g_3, \\ \phi_{4;K^*}^{\parallel(\perp)} &= \mathbb{A}_{(T)}, & \phi_{3;K^*}^{\perp} &= g_{\perp}^{(v)}, & \psi_{4;K^*}^{\perp} &= h_3. \end{aligned} \quad (2.11)$$

The conformal expansion of $\phi_2^{\parallel,\perp}$ reads

$$\phi_2^{\parallel,\perp}(u) = 6u\bar{u} \left\{ 1 + \sum_{n=1}^{\infty} a_n^{\parallel,\perp} C_n^{3/2}(2u-1) \right\} \quad (2.12)$$

in terms of the (non-perturbative) Gegenbauer moments $a_n^{\parallel,\perp}$ and the Gegenbauer polynomials $C_n^{3/2}$. To leading-logarithmic accuracy, the a_n renormalise multiplicatively as

$$a_n^{\text{LO}}(\mu^2) = L^{\gamma_n^{(0)}/(2\beta_0)} a_n(\mu_0^2), \quad (2.13)$$

where $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$, $\beta_0 = (33 - 2N_f)/3$, and the anomalous dimensions $\gamma_n^{(0)}$ are given by [19]

$$\begin{aligned} \gamma_n^{\parallel(0)} &= 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} - \frac{1}{2(n+1)(n+2)} \right), \\ \gamma_n^{\perp(0)} &= 8C_F \left(\psi(n+2) + \gamma_E - \frac{3}{4} \right). \end{aligned}$$

To next-to-leading order accuracy, the scale dependence of the Gegenbauer moments is more complicated and reads [20]

$$a_n^{\text{NLO}}(\mu^2) = a_n(\mu_0^2) E_n^{\text{NLO}} + \frac{\alpha_s}{4\pi} \sum_{k=0}^{n-2} a_k(\mu_0^2) L^{\gamma_k^{(0)}/(2\beta_0)} d_{nk}^{(1)}, \quad (2.14)$$

where

$$E_n^{\text{NLO}} = L^{\gamma_n^{(0)}/(2\beta_0)} \left\{ 1 + \frac{\gamma_n^{(1)}\beta_0 - \gamma_n^{(0)}\beta_1}{8\pi\beta_0^2} [\alpha_s(\mu^2) - \alpha_s(\mu_0^2)] \right\}$$

with $\beta_1 = 102 - (38/3)N_f$; $\gamma_n^{(1)}$ are the diagonal two-loop anomalous dimensions, which have been calculated, for the vector current, in ref. [21], and, for the tensor current, in ref. [22]. The mixing coefficients $d_{nk}^{(1)}$, $k \leq n-2$, are given, in closed form in ref. [20], for the axial vector current; the formulas are valid for arbitrary currents upon substitution of the corresponding one-loop anomalous dimension.

For the lowest moments $n = 0, 1, 2$ one has, explicitly:

$$\begin{aligned} \gamma_0^{\parallel(1)} &= 0, & \gamma_1^{\parallel(1)} &= \frac{23110}{243} - \frac{512}{81} N_f, & \gamma_2^{\parallel(1)} &= \frac{34072}{243} - \frac{830}{81} N_f, \\ \gamma_0^{\perp(1)} &= \frac{724}{9} - \frac{104}{27} N_f, & \gamma_1^{\perp(1)} &= 124 - 8N_f, & \gamma_2^{\perp(1)} &= \frac{38044}{243} - \frac{904}{81} N_f, \end{aligned} \quad (2.15)$$

and

$$\begin{aligned} d_{20}^{\parallel(1)} &= \frac{35}{9} \frac{20 - 3\beta_0}{50 - 9\beta_0} \left(1 - L^{50/(9\beta_0)-1} \right), \\ d_{20}^{\perp(1)} &= \frac{28}{9} \frac{16 - 3\beta_0}{40 - 9\beta_0} \left(1 - L^{40/(9\beta_0)-1} \right). \end{aligned} \quad (2.16)$$

Let us now review the numerical values of the twist-2 parameters, to NLO in conformal spin. The longitudinal decay constants of the charged mesons ρ^\pm , $K^{*\pm}$ can be extracted from the branching ratios of $\tau^- \rightarrow V^- \nu_\tau$, whereas $f_{\rho,\omega,\phi}^\parallel$ follow from $e^+e^- \rightarrow V^0$. A critical discussion of the results, including effects of ρ - ω and ω - ϕ mixing, was given in ref. [2] from which we quote the following results:

$$f_\rho^\parallel = (216 \pm 3) \text{ MeV}, \quad f_\omega^\parallel = (187 \pm 5) \text{ MeV}, \quad f_{K^*}^\parallel = (220 \pm 5) \text{ MeV}, \quad f_\phi^\parallel = (215 \pm 5) \text{ MeV}. \quad (2.17)$$

The transverse decay constants f^\perp , on the other hand, cannot be determined from experiment, but have to be calculated using non-perturbative methods. Currently available results include QCD sum rule determinations [9, 11, 15, 17] and lattice calculations [23]. The corresponding results have been critically reviewed and averaged in ref. [2], with the following results:

$$f_\rho^\perp = (165 \pm 9) \text{ MeV}, \quad f_\omega^\perp = (151 \pm 9) \text{ MeV}, \quad f_{K^*}^\perp = (185 \pm 10) \text{ MeV}, \quad f_\phi^\perp = (186 \pm 9) \text{ MeV}. \quad (2.18)$$

Let us now turn to the Gegenbauer moments $a_{1,2}^{\parallel,\perp}$. At present, there are no lattice determinations for any of those, so all available determinations come from QCD sum rules [11, 15–17, 24] or quark models [25]. For mesons with definite G parity (equal mass quarks), i.e. ρ and ϕ in our case, $a_1^{\parallel,\perp} = 0$. For $a_1^{\parallel,\perp}(K^*)$, the results from QCD sum rule calculations converge to [17]

$$\begin{aligned} a_1^\parallel(K^*)^{\mu=1 \text{ GeV}} &= 0.03 \pm 0.02, & a_1^\parallel(K^*)^{\mu=2 \text{ GeV}} &= 0.02 \pm 0.02, \\ a_1^\perp(K^*)^{\mu=1 \text{ GeV}} &= 0.04 \pm 0.03, & a_1^\perp(K^*)^{\mu=2 \text{ GeV}} &= 0.03 \pm 0.03. \end{aligned} \quad (2.19)$$

As for a_2 , $a_2^{\perp,\parallel}(\rho)$ have been determined in ref. [9] and reinvestigated, recently, in ref. [1], using the updated hadronic input collected in table 2. The resulting values

$$\begin{aligned} a_2^\parallel(\rho)^{\mu=1 \text{ GeV}} &= 0.15 \pm 0.07, & a_2^\parallel(\rho)^{\mu=2 \text{ GeV}} &= 0.10 \pm 0.05, \\ a_2^\perp(\rho)^{\mu=1 \text{ GeV}} &= 0.14 \pm 0.06, & a_2^\perp(\rho)^{\mu=2 \text{ GeV}} &= 0.11 \pm 0.05, \end{aligned} \quad (2.20)$$

are slightly smaller than those quoted in ref. [9]. The value of $a_2^{\parallel,\perp}(K^*)$ has been determined in ref. [11, 15] and reinvestigated in ref. [1]. The result is

$$\begin{aligned} a_2^\parallel(K^*)^{\mu=1 \text{ GeV}} &= 0.11 \pm 0.09, & a_2^\parallel(K^*)^{\mu=2 \text{ GeV}} &= 0.08 \pm 0.06, \\ a_2^\perp(K^*)^{\mu=1 \text{ GeV}} &= 0.10 \pm 0.08, & a_2^\perp(K^*)^{\mu=2 \text{ GeV}} &= 0.08 \pm 0.06. \end{aligned} \quad (2.21)$$

The corresponding parameters of the ϕ have received far less attention: ref. [11] quotes $a_2^{\parallel,\perp}(\phi)^{\mu=1 \text{ GeV}} = 0 \pm 0.1$ and ref. [2] $a_2^\perp(\phi)^{\mu=1 \text{ GeV}} = 0.2 \pm 0.2$. In this paper, we evaluate the sum rules collected in appendix A, which include all relevant corrections in m_s^2 and in particular the radiative corrections to the quark condensate term in $m_s \langle \bar{s}s \rangle$, and find

$$\begin{aligned} a_2^\parallel(\phi)^{\mu=1 \text{ GeV}} &= 0.18 \pm 0.08, & a_2^\parallel(\phi)^{\mu=2 \text{ GeV}} &= 0.13 \pm 0.06, \\ a_2^\perp(\phi)^{\mu=1 \text{ GeV}} &= 0.14 \pm 0.07, & a_2^\perp(\phi)^{\mu=2 \text{ GeV}} &= 0.11 \pm 0.05. \end{aligned} \quad (2.22)$$

In summary, it is probably fair to say that all known determinations of $a_{1,2}^{\perp,\parallel}$ point at fairly small values at 1 GeV and that within the present accuracy $a_{1(2)}^\perp = a_{1(2)}^\parallel$.

Let us now turn to twist-3 DAs.

3. Twist-3 distributions

To twist-3 accuracy, there is a total of four two-particle DAs and three three-particle DAs whose mutual interrelations have been unravelled in ref. [11], including quark mass corrections. The crucial point in constructing higher-twist DAs is the necessity to satisfy the QCD EOM which yield relations between physical effects of different origin: for example, using EOM, the contributions of orbital angular momentum in the valence component of the wave function can be expressed in terms of contributions of higher Fock states. An appropriate framework for implementing these constraints was developed in ref. [14]: it is based on the derivation of EOM relations for non-local light-ray operators [26], which are solved order by order in the conformal expansion; see ref. [12] for a review and further references. In this way one can construct self-consistent approximations for the DAs, which involve a minimum number of hadronic parameters. The EOM relations relating twist-2 and -3 DAs of vector mesons were derived in ref. [11], including all quark-mass corrections. What is new in the present paper is the inclusion of G-parity breaking corrections to three-particle DAs, which, via the EOM relations, also impact on the two-particle DAs. Based on these relations, we derive, in this section, complete formulas for all twist-3 DAs to NLO in the conformal expansion, including all G-parity breaking effects. A non-zero quark mass also induces a mixing of twist-2 parameters into those of twist-3 under a change of the renormalisation scale. We also derive the corresponding scaling relations.

Let us start by defining the relevant DAs. The two-particle twist-3 DAs $\phi_{3;K^*}^{\perp,\parallel}$ have already been defined in eqs. (2.9) and (2.9). There are two more two-particle DAs, $\psi_{3;K^*}^{\perp,\parallel}$, defined as⁴

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5s(-z)|K^*(P,\lambda)\rangle = \frac{1}{2}f_{K^*}^{\parallel}m_{K^*}\epsilon_{\mu\nu\alpha\beta}e_\nu^{\nu\alpha\beta}e_\nu^{(\lambda)}p_\alpha z_\beta \int_0^1 du e^{i\xi px}\psi_{3;K^*}^\perp(u), \quad (3.1)$$

$$\langle 0|\bar{q}(z)s(-z)|K^*(P,\lambda)\rangle = -if_{K^*}^\perp(e^{(\lambda)}z)m_{K^*}^2 \int_0^1 du e^{i\xi pz}\psi_{3;K^*}^\parallel(u). \quad (3.2)$$

The normalisation is given by

$$\int_0^1 du \psi_{3;K^*}^{\parallel(\perp)}(u) = 1 - \frac{f_{K^*}^{\parallel(\perp)}}{f_{K^*}^\perp} \frac{m_s + m_q}{m_{K^*}}, \quad (3.3)$$

which differs from ref. [11], where all DAs were normalised to 1. The reason is that in [11] we implicitly expanded the normalisation factor $1/\{1 - (f_{K^*}^{\parallel(\perp)}/f_{K^*}^\perp)(m_s + m_q)/m_{K^*}\}$ in powers of $m_s + m_q$, whereas in this paper we keep the full dependence on the quark masses.

There are also three three-particle DAs of twist 3:

$$\begin{aligned} \langle 0|\bar{q}(z)g\tilde{G}_{\beta z}(vz)\gamma_z\gamma_5s(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^{\parallel}m_{K^*}(pz)^2 e_{\perp\beta}^{(\lambda)}\tilde{\Phi}_{3;K}^{\parallel}(v,pz) + \dots, \\ \langle 0|\bar{q}(z)gG_{\beta z}(vz)i\gamma_zs(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^{\parallel}m_{K^*}(pz)^2 e_{\perp\beta}^{(\lambda)}\Phi_{3;K}^{\parallel}(v,pz) + \dots, \\ \langle 0|\bar{q}(z)gG_{z\beta}(vz)\sigma_{z\beta}s(-z)|K^*(P,\lambda)\rangle &= f_{K^*}^\perp m_{K^*}^2(e^{(\lambda)}z)(pz)\Phi_{3;K^*}^\perp(v,pz), \end{aligned} \quad (3.4)$$

⁴In the notations of ref. [11], $\psi_{3;K^*}^\perp = \{1 - (f_{K^*}^\perp/f_{K^*}^{\parallel})\}(m_s + m_q)/m_{K^*}\}g_\perp^{(a)}$, $\psi_{3;K^*}^\parallel = \{1 - (f_{K^*}^\parallel/f_{K^*}^\perp)\}(m_s + m_q)/m_{K^*}\}h_\parallel^{(s)}$.

where the dots denote terms of higher twist and we use the short-hand notation

$$\mathcal{F}(v, pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_2 - \alpha_1 + v\alpha_3)} \mathcal{F}(\underline{\alpha}) \quad (3.5)$$

with $\mathcal{F}(\underline{\alpha})$ being a three-particle DA. $\underline{\alpha}$ is the set of parton momentum fractions $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$ and the integration measure $\mathcal{D}\underline{\alpha}$ is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \sum \alpha_i). \quad (3.6)$$

As discussed in ref. [11], all these DAs are interconnected by the QCD EOM. The analysis of these EOM and the resulting relations including quark mass corrections in $m_s \pm m_q$ is the subject of ref. [11], so we do not repeat it here, but just quote the results:

$$\begin{aligned} \psi_{3;K^*}^{\parallel}(u) &= \bar{u} \int_0^u dv \frac{1}{v} \Upsilon(v) + u \int_u^1 dv \frac{1}{v} \Upsilon(v), \\ \phi_{3;K^*}^{\parallel}(u) &= \frac{1}{2} \xi \left[\int_0^u dv \frac{1}{v} \Upsilon(v) - \int_u^1 dv \frac{1}{v} \Upsilon(v) \right] + \frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}} \frac{m_s + m_q}{m_{K^*}} \phi_{2;K^*}^{\parallel}(u) \\ &\quad + \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \Phi_{3;K^*}^{\perp}(\underline{\alpha}) \end{aligned} \quad (3.7)$$

with

$$\begin{aligned} \Upsilon(u) &= 2\phi_{2;K^*}^{\perp}(u) - \frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}} \frac{m_s + m_q}{m_{K^*}} \left[1 - \frac{1}{2} \xi \frac{d}{du} \right] \phi_{2;K^*}^{\parallel}(u) - \frac{1}{2} \frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}} \frac{m_s - m_q}{m_{K^*}} \frac{d}{du} \phi_{2;K^*}^{\parallel}(u) \\ &\quad + \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \left(\alpha_1 \frac{d}{d\alpha_1} + \alpha_2 \frac{d}{d\alpha_2} - 1 \right) \Phi_{3;K^*}^{\perp}(\underline{\alpha}) \end{aligned} \quad (3.8)$$

and

$$\begin{aligned} \psi_{3;K^*}^{\perp}(u) &= \bar{u} \int_0^u dv \frac{1}{v} \Omega(v) + u \int_u^1 dv \frac{1}{v} \Omega(v), \\ \phi_{3;K^*}^{\perp}(u) &= \frac{1}{4} \left[\int_0^u dv \frac{1}{v} \Omega(v) + \int_u^1 dv \frac{1}{v} \Omega(v) \right] + \frac{f_{K^*}^{\perp}}{f_{K^*}^{\parallel}} \frac{m_s + m_q}{m_{K^*}} \phi_{2;K^*}^{\perp}(u) \\ &\quad + \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \Phi_{3;K^*}^{\parallel}(\underline{\alpha}) \\ &\quad + \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \left(\frac{d}{d\alpha_1} + \frac{d}{d\alpha_2} \right) \tilde{\Phi}_{3;K^*}^{\parallel}(\underline{\alpha}) \end{aligned} \quad (3.9)$$

with

$$\begin{aligned} \Omega(u) &= 2\phi_{2;K^*}^{\parallel}(u) + \frac{f_{K^*}^{\perp}}{f_{K^*}^{\parallel}} \frac{m_s + m_q}{m_{K^*}} \xi \frac{d}{du} \phi_{2;K^*}^{\perp}(u) - \frac{f_{K^*}^{\perp}}{f_{K^*}^{\parallel}} \frac{m_s - m_q}{m_{K^*}} \frac{d}{du} \phi_{2;K^*}^{\perp}(u) \\ &\quad + 2 \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \left(\alpha_1 \frac{d}{d\alpha_1} + \alpha_2 \frac{d}{d\alpha_2} \right) \Phi_{3;K^*}^{\parallel}(\underline{\alpha}) \\ &\quad + 2 \frac{d}{du} \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{1}{\alpha_3} \left(\alpha_1 \frac{d}{d\alpha_1} - \alpha_2 \frac{d}{d\alpha_2} \right) \tilde{\Phi}_{3;K^*}^{\parallel}(\underline{\alpha}). \end{aligned} \quad (3.10)$$

The twist-3 three-particle DAs correspond to the light-cone projection $\gamma_z G_{z\perp}$ and $\sigma_{\perp z} G_{\perp z}$, respectively, which picks up the $s = \frac{1}{2}$ component of the quark fields and the $s = 1$ component of the gluonic field strength tensor. According to (2.7), the (normalised) asymptotic DA is then given by $360\alpha_1\alpha_2\alpha_3^2$. To NLO in the conformal expansion, each three-particle twist-3 DA involves three hadronic parameters, which we label in the following way: ζ, κ are LO and ω, λ NLO parameters. ζ and ω are G-parity conserving, whereas κ and λ violate G-parity and hence vanish for mesons with quarks of equal mass, i.e. ρ and ϕ . We then have

$$\begin{aligned}\Phi_{3;K^*}^{\parallel}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{ \kappa_{3K^*}^{\parallel} + \omega_{3K^*}^{\parallel}(\alpha_1 - \alpha_2) + \lambda_{3K^*}^{\parallel} \frac{1}{2} (7\alpha_3 - 3) \right\}, \\ \tilde{\Phi}_{3;K^*}^{\parallel}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{ \zeta_{3K^*}^{\parallel} + \tilde{\lambda}_{3K^*}^{\parallel}(\alpha_1 - \alpha_2) + \tilde{\omega}_{3K^*}^{\parallel} \frac{1}{2} (7\alpha_3 - 3) \right\}, \\ \Phi_{3;K^*}^{\perp}(\underline{\alpha}) &= 360\alpha_1\alpha_2\alpha_3^2 \left\{ \kappa_{3K^*}^{\perp} + \omega_{3K^*}^{\perp}(\alpha_1 - \alpha_2) + \lambda_{3K^*}^{\perp} \frac{1}{2} (7\alpha_3 - 3) \right\}.\end{aligned}\quad (3.11)$$

The relation to the parameters used in ref. [11] is $\zeta_3^A = \zeta_3^{\parallel}$, $\zeta_3^V = \omega_3^{\parallel}/14$, $\zeta_3^T = \omega_3^{\perp}/14$, $\zeta_3^{\parallel}\omega_{1,0}^A = \tilde{\omega}_3^{\parallel}$; G-parity breaking terms were not considered in ref. [11]. For equal mass quarks, $\Phi_{3;K^*}^{\perp}$ are antisymmetric under $\alpha_1 \leftrightarrow \alpha_2$, whereas $\tilde{\Phi}_{3;K^*}^{\parallel}$ is symmetric.

All these parameters can be defined in terms of matrix elements of local twist-3 operators. For chiral-odd operators, for instance, one has

$$\begin{aligned}\langle 0|\bar{q}\sigma_{z\xi}gG_{z\xi}s|K^*(P,\lambda)\rangle &= f_{K^*}^{\perp}m_{K^*}^2(e^{(\lambda)}z)(pz)\kappa_{3K^*}^{\perp}, \quad (3.12) \\ \langle 0|\bar{q}\sigma_{z\xi}[iD_z, gG_{z\xi}]s - \frac{3}{7}i\partial_z\bar{q}\sigma_{z\xi}gG_{z\xi}s|K^*(P,\lambda)\rangle &= f_{K^*}^{\perp}m_{K^*}^2(e^{(\lambda)}z)(pz)^2\frac{3}{28}\lambda_{3K^*}^{\perp}, \\ \langle 0|\bar{q}i\overleftarrow{D}_z\sigma_{z\xi}gG_{z\xi}s - \bar{q}\sigma_{z\xi}gG_{z\xi}i\overrightarrow{D}_zs|K^*(P,\lambda)\rangle &= f_{K^*}^{\perp}m_{K^*}^2(e^{(\lambda)}z)(pz)^2\frac{1}{14}\omega_{3K^*}^{\perp};\end{aligned}$$

the formulas for chiral-even operators are analogous. Numerical values for these parameters can be obtained from QCD sum rules and will be discussed in section 4.

Of the parameters in (3.11), $\zeta_{3K^*}^{\parallel}$, $\kappa_{3K^*}^{\perp}$, $\omega_{3K^*}^{\perp}$ and $\lambda_{3K^*}^{\perp}$ renormalise multiplicatively in the chiral limit, the others mix with each other. For non-zero strange quark mass, there is additional mixing with twist-2 parameters. Here, we write down explicitly only the RG-improved relations for the above 5 parameters; a full discussion, including also λ_3^{\parallel} , ω_3^{\parallel} , is given in appendix A. The relations can be written in compact form as

$$P_i(\mu^2) = L^{(\gamma_P)_i/\beta_0} P_i(\mu_0^2) + \sum_{j=1}^3 C_{ij} \left(L^{(\gamma_Q)_{ij}/\beta_0} - L^{(\gamma_P)_i/\beta_0} \right) Q_{ij}(\mu_0^2) \quad (3.13)$$

with the LO scaling factor $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$. If there is a flavour threshold μ_{th} between μ_0 and μ changing the number of active flavours from n_f to $n_f + 1$, then one has to replace

$$L^{1/\beta_0} \rightarrow (\alpha(\mu^2)/\alpha(\mu_{\text{th}}^2))^{1/\beta_0(n_f+1)} (\alpha(\mu_{\text{th}}^2)/\alpha(\mu_0^2))^{1/\beta_0(n_f)}.$$

The parameters in (3.13) are given by:

$$\begin{aligned}
 P &= \{f_{K^*}^{\parallel} \zeta_{3K^*}^{\parallel}, f_{K^*}^{\parallel} \kappa_{3K^*}^{\parallel}, f_{K^*}^{\perp} \kappa_{3K^*}^{\perp}, f_{K^*}^{\perp} \omega_{3K^*}^{\perp}, f_{K^*}^{\perp} \lambda_{3K^*}^{\perp}\}, \\
 Q_{1(2)} &= \frac{f_{K^*}^{\perp}}{m_{K^*}} \{m_s \pm m_q, (m_s \mp m_q) a_1^{\perp}, (m_s \pm m_q) a_2^{\perp}\}, \\
 Q_{3,5} &= \frac{f_{K^*}^{\parallel}}{m_{K^*}} \{m_s - m_q, (m_s + m_q) a_1^{\parallel}, (m_s - m_q) a_2^{\parallel}\}, \\
 Q_4 &= \frac{f_{K^*}^{\parallel}}{m_{K^*}} \{m_s + m_q, (m_s - m_q) a_1^{\parallel}, (m_s + m_q) a_2^{\parallel}\}, \\
 \gamma_P &= \left\{ \frac{77}{9}, \frac{77}{9}, \frac{55}{9}, \frac{73}{9}, \frac{104}{9} \right\}, \\
 (\gamma_Q)_{1,2} &= \left\{ \frac{16}{3}, 8, \frac{88}{9} \right\}, \quad (\gamma_Q)_{3,4,5} = \left\{ 4, \frac{68}{9}, \frac{86}{9} \right\}, \\
 C &= \begin{pmatrix} \frac{2}{29} & \frac{6}{25} & 0 \\ -\frac{2}{29} & -\frac{6}{25} & 0 \\ -\frac{4}{19} & \frac{12}{65} & 0 \\ \frac{14}{37} & -\frac{42}{25} & \frac{12}{13} \\ -\frac{1}{85} & -\frac{1}{5} & -\frac{4}{15} \end{pmatrix}. \tag{3.14}
 \end{aligned}$$

Implicit formulas for the remaining 4 parameters in (3.11) can be found in appendix A. Numerical values will be given in the next section.

Using (3.11), and the corresponding relations for twist-2 DAs, one obtains expressions for the twist-3 two-particle DAs, which are valid to NLO in the conformal expansion. As discussed in ref. [11], the structure of this expansion is complicated by the fact that these DAs do not correspond to a fixed projection of the quark fields' Lorentz-spin s . The resulting expansion is in $C_n^{3/2}(\xi)$ for $\psi_{3;K^*}^{\perp,\parallel}$ and $C_n^{1/2}(\xi)$ for $\phi_{3;K^*}^{\perp,\parallel}$:

$$\begin{aligned}
 \phi_{3;K^*}^{\parallel}(u) &= 3\xi^2 + \frac{3}{2} \xi(3\xi^2 - 1) a_1^{\perp} + \frac{3}{2} \xi^2(5\xi^2 - 3) a_2^{\perp} \\
 &+ \left(\frac{15}{2} \kappa_{3K^*}^{\perp} - \frac{3}{4} \lambda_{3K^*}^{\perp} \right) \xi(5\xi^2 - 3) + \frac{5}{8} \omega_{3K^*}^{\perp} (3 - 30\xi^2 + 35\xi^4) \\
 &+ \frac{3}{2} \frac{m_s + m_q}{m_{K^*}} \frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}} \left\{ 1 + 8\xi a_1^{\parallel} + 3(7 - 30u\bar{u}) a_2^{\parallel} + \xi \ln \bar{u} (1 + 3a_1^{\parallel} + 6a_2^{\parallel}) \right. \\
 &\quad \left. - \xi \ln u (1 - 3a_1^{\parallel} + 6a_2^{\parallel}) \right\} \\
 &- \frac{3}{2} \frac{m_s - m_q}{m_{K^*}} \frac{f_{K^*}^{\parallel}}{f_{K^*}^{\perp}} \xi \left\{ 2 + 9\xi a_1^{\parallel} + 2(11 - 30u\bar{u}) a_2^{\parallel} + \ln \bar{u} (1 + 3a_1^{\parallel} + 6a_2^{\parallel}) \right. \\
 &\quad \left. + \ln u (1 - 3a_1^{\parallel} + 6a_2^{\parallel}) \right\}, \tag{3.15}
 \end{aligned}$$

$$\begin{aligned}
 \psi_{3;K^*}^{\parallel}(u) = & 6u\bar{u} \left\{ 1 + \left(\frac{a_1^\perp}{3} + \frac{5}{3}\kappa_{3K^*}^\perp \right) C_1^{3/2}(\xi) + \left(\frac{a_2^\perp}{6} + \frac{5}{18}\omega_{3K^*}^\perp \right) C_2^{3/2}(\xi) - \frac{1}{20}\lambda_{3K^*}^\perp C_3^{3/2}(\xi) \right\} \\
 & + 3 \frac{m_s + m_q}{m_{K^*}} \frac{f_{K^*}^{\parallel}}{f_{K^*}^\perp} \left\{ u\bar{u}(1 + 2\xi a_1^\parallel + 3(7 - 5u\bar{u})a_2^\parallel) + \bar{u} \ln \bar{u}(1 + 3a_1^\parallel + 6a_2^\parallel) \right. \\
 & \quad \left. + u \ln u(1 - 3a_1^\parallel + 6a_2^\parallel) \right\} \\
 & - 3 \frac{m_s - m_q}{m_{K^*}} \frac{f_{K^*}^{\parallel}}{f_{K^*}^\perp} \left\{ u\bar{u}(9a_1^\parallel + 10\xi a_2^\parallel) + \bar{u} \ln \bar{u}(1 + 3a_1^\parallel + 6a_2^\parallel) \right. \\
 & \quad \left. - u \ln u(1 - 3a_1^\parallel + 6a_2^\parallel) \right\}, \tag{3.16}
 \end{aligned}$$

$$\begin{aligned}
 \psi_{3;K^*}^\perp(u) = & 6u\bar{u} \left\{ 1 + \left(\frac{1}{3}a_1^\parallel + \frac{20}{9}\kappa_{3K^*}^\parallel \right) C_1^{3/2}(\xi) \right. \\
 & \left. + \left(\frac{1}{6}a_2^\parallel + \frac{10}{9}\zeta_{3K^*}^\parallel + \frac{5}{12}\omega_{3K^*}^\parallel - \frac{5}{24}\tilde{\omega}_{3K^*}^\parallel \right) C_2^{3/2}(\xi) + \left(\frac{1}{4}\tilde{\lambda}_{3K^*}^\parallel - \frac{1}{8}\lambda_{3K^*}^\parallel \right) C_3^{3/2}(\xi) \right\} \\
 & + 6 \frac{m_s + m_q}{m_{K^*}} \frac{f_{K^*}^\perp}{f_{K^*}^\parallel} \left\{ u\bar{u}(2 + 3\xi a_1^\perp + 2(11 - 10u\bar{u})a_2^\perp) + \bar{u} \ln \bar{u}(1 + 3a_1^\perp + 6a_2^\perp) \right. \\
 & \quad \left. + u \ln u(1 - 3a_1^\perp + 6a_2^\perp) \right\} \\
 & - 6 \frac{m_s - m_q}{m_{K^*}} \frac{f_{K^*}^\perp}{f_{K^*}^\parallel} \left\{ u\bar{u}(9a_1^\perp + 10\xi a_2^\perp) + \bar{u} \ln \bar{u}(1 + 3a_1^\perp + 6a_2^\perp) \right. \\
 & \quad \left. - u \ln u(1 - 3a_1^\perp + 6a_2^\perp) \right\}, \tag{3.17}
 \end{aligned}$$

$$\begin{aligned}
 \phi_{3;K^*}^\perp(u) = & \frac{3}{4}(1 + \xi^2) + \frac{3}{2}\xi^3 a_1^\parallel + \left\{ \frac{3}{7}a_2^\parallel + 5\zeta_{3K^*}^\parallel \right\} (3\xi^2 - 1) + \left\{ 5\kappa_{3K^*}^\parallel - \frac{15}{16}\lambda_{3K^*}^\parallel \right. \\
 & \left. + \frac{15}{8}\tilde{\lambda}_{3K^*}^\parallel \right\} \xi(5\xi^2 - 3) + \left\{ \frac{9}{112}a_2^\parallel + \frac{15}{32}\omega_{3K^*}^\parallel - \frac{15}{64}\tilde{\omega}_{3K^*}^\parallel \right\} (35\xi^4 - 30\xi^2 + 3) \\
 & + \frac{3}{2} \frac{m_s + m_q}{m_{K^*}} \frac{f_{K^*}^\perp}{f_{K^*}^\parallel} \left\{ 2 + 9\xi a_1^\perp + 2(11 - 30u\bar{u})a_2^\perp \right. \\
 & \quad \left. + (1 - 3a_1^\perp + 6a_2^\perp) \ln u + (1 + 3a_1^\perp + 6a_2^\perp) \ln \bar{u} \right\} \\
 & - \frac{3}{2} \frac{m_s - m_q}{m_{K^*}} \frac{f_{K^*}^\perp}{f_{K^*}^\parallel} \left\{ 2\xi + 9(1 - 2u\bar{u})a_1^\perp + 2\xi(11 - 20u\bar{u})a_2^\perp \right. \\
 & \quad \left. + (1 + 3a_1^\perp + 6a_2^\perp) \ln \bar{u} - (1 - 3a_1^\perp + 6a_2^\perp) \ln u \right\}. \tag{3.18}
 \end{aligned}$$

These expressions are our final results for the two-particle twist-3 DAs and supersede those given in ref. [11] where G-parity violating terms in κ_3 and λ_3 were not included. The terms multiplying $\ln u$ and $\ln \bar{u}$ are the first three terms in the conformal expansion of $d/(du)\phi_{2;K^*}^{\parallel,\perp}(0)$ and $d/(du)\phi_{2;K^*}^{\parallel,\perp}(1)$, respectively. Numerical values for the hadronic parameters are given in table 1.

4. Models for distribution amplitudes

In this section we compile the numerical estimates of all necessary parameters and present

	ρ		K^*		ϕ	
	$\mu = 1 \text{ GeV}$	$\mu = 2 \text{ GeV}$	$\mu = 1 \text{ GeV}$	$\mu = 2 \text{ GeV}$	$\mu = 1 \text{ GeV}$	$\mu = 2 \text{ GeV}$
a_1^{\parallel}	0	0	0.03(2)	0.02(2)	0	0
a_1^{\perp}	0	0	0.04(3)	0.03(3)	0	0
a_2^{\parallel}	0.15(7)	0.10(5)	0.11(9)	0.08(6)	0.18(8)	0.13(6)
a_2^{\perp}	0.14(6)	0.11(5)	0.10(8)	0.08(6)	0.14(7)	0.11(5)
ζ_{3V}^{\parallel}	0.030(10)	0.020(9)	0.023(8)	0.015(6)	0.024(8)	0.017(6)
$\tilde{\lambda}_{3V}^{\parallel}$	0	0	0.035(15)	0.017(8)	0	0
$\tilde{\omega}_{3V}^{\parallel}$	-0.09(3)	-0.04(2)	-0.07(3)	-0.03(2)	-0.045(15)	-0.022(8)
κ_{3V}^{\parallel}	0	0	0.000(1)	-0.001(2)	0	0
ω_{3V}^{\parallel}	0.15(5)	0.09(3)	0.10(4)	0.06(3)	0.09(3)	0.06(2)
λ_{3V}^{\parallel}	0	0	-0.008(4)	-0.004(2)	0	0
κ_{3V}^{\perp}	0	0	0.003(3)	-0.001(2)	0	0
ω_{3V}^{\perp}	0.55(25)	0.37(19)	0.3(1)	0.2(1)	0.20(8)	0.15(7)
λ_{3V}^{\perp}	0	0	-0.025(20)	-0.015(10)	0	0

Table 1: Twist-2 and -3 hadronic parameters at the scale $\mu = 1 \text{ GeV}$ and scaled to up $\mu = 2 \text{ GeV}$, using the evolution equations (3.13). The sign of the twist-3 parameters corresponds to the sign convention for the strong coupling defined by the covariant derivative $D_\mu = \partial_\mu - igA_\mu^a t^a$; they change sign if g is fixed by $D_\mu = \partial_\mu + igA_\mu^a t^a$.

explicit models of the twist-3 two-particle distribution amplitudes introduced in the last section. The important point is that these DAs are related to three-particle ones by exact QCD equations of motion and have to be used together; this guarantees the consistency of the approximation. Our approximation thus introduces a minimum number of non-perturbative parameters, which are defined as matrix elements of certain local operators between the vacuum and the meson state, and which we estimate using QCD sum rules. More sophisticated models can be constructed in a systematic way by adding contributions of higher conformal partial waves when estimates of the relevant non-perturbative matrix elements will become available.

Our approach involves the implicit assumption that the conformal partial wave expansion is well convergent. This can be justified rigorously at large scales, since the anomalous dimensions of all involved operators increase logarithmically with the conformal spin J , but is non-trivial at relatively low scales of order $\mu \sim (1-2) \text{ GeV}$ which we choose as reference scale.

Since orthogonal polynomials of high orders are rapidly oscillating functions, a truncated expansion in conformal partial waves is, almost necessarily, oscillatory as well. Such a behaviour is clearly unphysical, but this does not constitute a real problem since physical observables are given by convolution integrals of distribution amplitudes with smooth co-

efficient functions. A classical example for this feature is the $\gamma\gamma^*$ -meson form factor, which is governed by the quantity

$$\int du \frac{1}{u} \phi(u) \sim \sum a_i,$$

where the coefficients a_i are exactly the “reduced matrix elements” in the conformal expansion. The oscillating terms are averaged over and strongly suppressed. Stated otherwise: models of distribution amplitudes should generally be understood as distributions (in the mathematical sense).

We give all relevant numerical input parameters for our model DAs in table 1, at the scale $\mu = 1 \text{ GeV}$, which is appropriate for QCD sum-rule results, and, using the LO and NLO scaling relations given in sections 2.3 and 3, at the scale $\mu = 2 \text{ GeV}$, in order to facilitate the comparison with future lattice determinations of these quantities. The mixing of K^* and ϕ parameters with operators of lower twist depending on m_s is numerically small. In evaluating the sum rules, we have chosen the values of the continuum threshold s_0 as given in appendix C. The sum rules are actually rather insensitive to that parameter, due to the smallness of the perturbative contribution, but are not very stable in the Borel parameter M^2 , which is the reason for the large uncertainties in table 1. The biggest contribution to G-conserving parameters comes from the gluon condensate. For G-breaking parameters, on the other hand, this contribution is suppressed by a factor m_s^2 and, as a consequence, all G-breaking parameters are considerably smaller than the G-conserving ones. The table also shows that SU(3) breaking is relevant for all parameters.

At this point we would like to compare our results with those available in the literature. We have discussed the twist-2 parameters already in section 2.3. As for twist-3, only results for ρ are available. A long time ago, the chiral-even parameters were determined, in ref. [27], as

$$\zeta_{3\rho}^{\parallel}(1 \text{ GeV}) = 0.033 \pm 0.003, \quad \omega_{3\rho}^{\parallel}(1 \text{ GeV}) = 0.2, \quad \tilde{\omega}_{3\rho}^{\parallel}(1 \text{ GeV}) = -0.1. \quad (4.1)$$

A comparison with table 1 shows that these values agree quite well with ours, although we think that the uncertainty of $\zeta_{3\rho}^{\parallel}$ was underestimated in [27]. A value for $\omega_{3\rho}^{\perp}$ was obtained in ref. [11]: $\omega_{3\rho}^{\perp}(1 \text{ GeV}) = 0.3 \pm 0.3$, which is a bit smaller than our result.

In figure 1 we plot the longitudinal twist-3 two-particle DAs ϕ_3^{\parallel} and ψ_3^{\parallel} for the ρ meson, assuming massless quarks, and for the K^* and ϕ mesons, together with the corresponding asymptotic DAs. Figure 2 shows the transverse DAs ϕ_3^{\perp} and ψ_3^{\perp} . The figures show that quark-mass corrections significantly modify the end-point behaviour of $\phi_3^{\parallel,\perp}$, where they induce a logarithmic end-point divergency, even if the contributions of gluonic operators are neglected. This is not a problem because, as mentioned above, the DAs themselves need not be finite, it is only their convolution with perturbative scattering amplitudes that is meaningful. The figures also show that the effect of SU(3) breaking (the difference between the red and the other curves) is quite pronounced for all DAs, whereas the G-parity breaking terms (the asymmetry of the green curve) have only minor impact, which is due to the numerical smallness of the corresponding hadronic parameters.

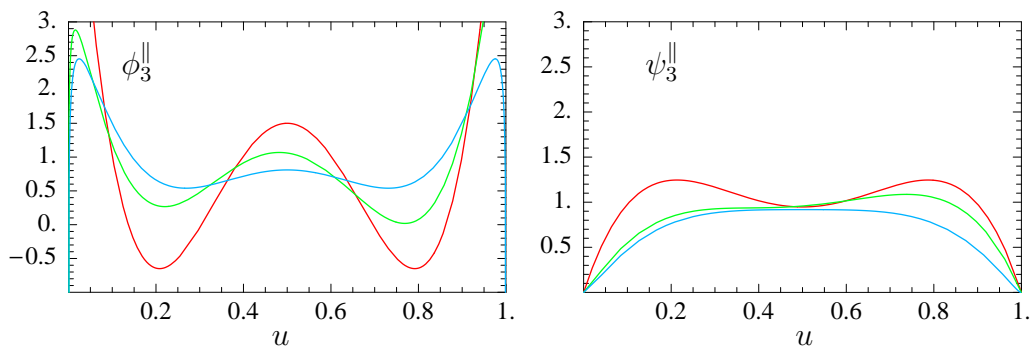


Figure 1: Left panel: ϕ_3^{\parallel} as a function of u for the central values of hadronic parameters, for $\mu = 1$ GeV. Red line: $\phi_{3;\rho}^{\parallel}$, green: $\phi_{3;K^*}^{\parallel}$, blue: $\phi_{3;\phi}^{\parallel}$. Right panel: same for ψ_3^{\parallel} .

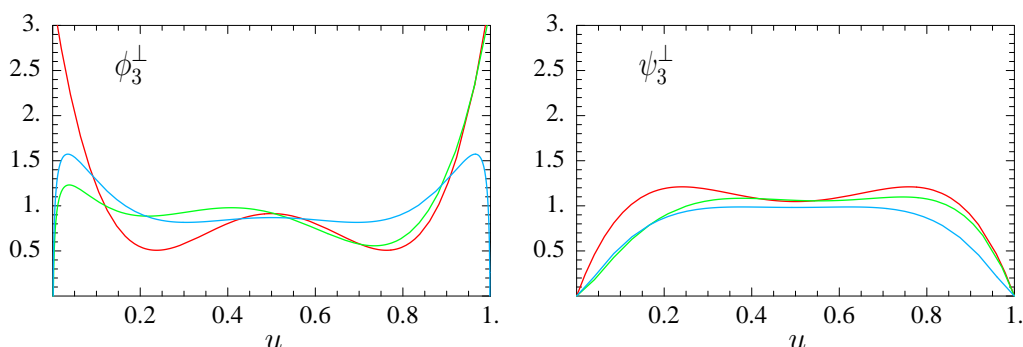


Figure 2: Left panel: ϕ_3^{\perp} as a function of u for the central values of hadronic parameters, for $\mu = 1$ GeV. Red line: $\phi_{3;\rho}^{\perp}$, green: $\phi_{3;K^*}^{\perp}$, blue: $\phi_{3;\phi}^{\perp}$. Right panel: same for ψ_3^{\perp} .

5. Summary and conclusions

In this paper we have studied the twist-3 two- and three-particle distribution amplitudes of K^* and ϕ mesons in QCD and expressed them in a model-independent way by a minimal number of non-perturbative parameters. The work presented here is an extension of refs. [7, 11] and completes the analysis of SU(3)-breaking in vector meson distribution amplitudes to twist-3 accuracy. Our approach consists of two components. One is the use of the QCD equations of motion, which allow dynamically dependent DAs to be expressed in terms of independent ones. The other ingredient is conformal expansion, which makes it possible to separate transverse and longitudinal variables in the wave functions, the former ones being governed by renormalisation-group equations, the latter ones being described in terms of irreducible representations of the corresponding symmetry group. We have derived expressions for all twist-3 two- and three-particle distribution amplitudes to next-to-leading order in the conformal expansion, including both chiral corrections $\mathcal{O}(m_s + m_q)$ and G-parity-breaking corrections $\mathcal{O}(m_s - m_q)$; the corresponding formulas are given in section 3.

We have also done a complete reanalysis of the numerical values of the relevant twist-3 hadronic parameters from QCD sum rules. Our sum rules can be compared, in the chiral limit, with existing calculations for the ρ [11, 27]. We have also studied the scale-dependence of all parameters to leading-logarithmic, or, if possible, next-to-leading-logarithmic accuracy, taking into account the mixing with operators depending on the strange-quark mass m_s . Our final numerical results, at the scales 1 and 2 GeV, are collected in table 1.

Preliminary versions of our results have already been applied in studies of $B \rightarrow (\rho, K^*)\gamma$ decays [1, 2, 8]. These processes are also sensitive to twist-4 distribution amplitudes which we will study in a separate publication. While the parametrisations given in section 3 are general, the actual numerical values for hadronic parameters given in section 2.3 and table 1 are obtained within the framework of QCD sum rules. We are looking forward to a confirmation of these values from independent non-perturbative methods, for instance lattice QCD.

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A. Scale-dependence of twist-3 parameters

In this appendix we derive the renormalisation-group improved expression (3.13) for twist-3 parameters and also give an implicit relation for the scaling of the chiral-even parameters ω_3 , $\tilde{\omega}_3$, λ_3 and $\tilde{\lambda}_3$.

Let us introduce the following notation for the relevant quark-quark-gluon operators, see eq. (3.11):

$$\begin{aligned} O_3^T(z, vz, -z) &= \bar{q}(z)\sigma_{z\nu}gG_{z\nu}(vz)s(-z), \\ O_3(z, vz, -z) &= \bar{q}(z)gG_{\perp z}(vz)i\gamma_zs(-z), \\ \tilde{O}_3(z, vz, -z) &= \bar{q}(z)g\tilde{G}_{\perp z}(vz)\gamma_z\gamma_5s(-z). \end{aligned} \tag{A.1}$$

In principle it is possible to establish evolution equations for these non-local operators using the light-ray operator technique developed in ref. [26]. The scale-dependence of the parameters in (3.11) then follows from a projection of the evolution equation on the corresponding conformal wave. Another approach is to make use of the results derived in the literature for the anomalous dimensions of moments of the corresponding nucleon structure functions. The explicit relations between these anomalous dimensions, given in ref. [28], and those needed for our twist-3 parameters have been established in ref. [11]. Neglecting quark mass corrections, one has, in the notations of ref. [11],

$$\begin{aligned} [f_{K^*}^\perp \kappa_{3K^*}^\perp](\mu^2) &= L^{\Gamma_2^{T^+}/\beta_0} [f_{K^*}^\perp \kappa_{3K^*}^\perp](\mu_0^2), \\ [f_{K^*}^\perp \omega_{3K^*}^\perp](\mu^2) &= L^{\Gamma_3^{T^+}/\beta_0} [f_{K^*}^\perp \omega_{3K^*}^\perp](\mu_0^2), \end{aligned}$$

$$\begin{aligned}
[f_{K^*}^\perp \lambda_{3K^*}^\perp](\mu^2) &= L^{\Gamma_3^{T^-}/\beta_0} [f_{K^*}^\perp \lambda_{3K^*}^\perp](\mu_0^2), \\
f_{K^*}^\parallel \zeta_{3K^*}^\parallel(\mu^2) &= L^{\Gamma_2^-/\beta_0} f_{K^*}^\parallel \zeta_{3K^*}^\parallel(\mu_0^2), \\
f_{K^*}^\parallel \kappa_{3K^*}^\parallel(\mu^2) &= L^{\Gamma_2^+/\beta_0} f_{K^*}^\parallel \kappa_{3K^*}^\parallel(\mu_0^2), \\
f_{K^*}^\parallel \left(\frac{2}{3} \omega_{3K^*}^\parallel - \tilde{\omega}_{3K^*}^\parallel \right)^{\mu^2} &= L^{\Gamma_3^+/\beta_0} f_{K^*}^\parallel \left(\frac{2}{3} \omega_{3K^*}^\parallel - \tilde{\omega}_{3K^*}^\parallel \right)^{\mu_0^2}, \\
f_{K^*}^\parallel \left(\frac{2}{3} \omega_{3K^*}^\parallel + \tilde{\omega}_{3K^*}^\parallel \right)^{\mu^2} &= L^{\Gamma_3^+/\beta_0} f_{K^*}^\parallel \left(\frac{2}{3} \omega_{3K^*}^\parallel + \tilde{\omega}_{3K^*}^\parallel \right)^{\mu_0^2}, \\
f_{K^*}^\parallel \left(\frac{2}{3} \tilde{\lambda}_{3K^*}^\parallel + \lambda_{3K^*}^\parallel \right)^{\mu^2} &= L^{\Gamma_3^-/\beta_0} f_{K^*}^\parallel \left(\frac{2}{3} \tilde{\lambda}_{3K^*}^\parallel + \lambda_{3K^*}^\parallel \right)^{\mu_0^2}, \\
f_{K^*}^\parallel \left(\frac{2}{3} \tilde{\lambda}_{3K^*}^\parallel - \lambda_{3K^*}^\parallel \right)^{\mu^2} &= L^{\Gamma_3^-/\beta_0} f_{K^*}^\parallel \left(\frac{2}{3} \tilde{\lambda}_{3K^*}^\parallel - \lambda_{3K^*}^\parallel \right)^{\mu_0^2}, \tag{A.2}
\end{aligned}$$

where L is the leading-log scaling factor $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$. The factor $2/3$ comes from the relative factors in (3.13). The anomalous dimensions are given by

$$\begin{aligned}
\Gamma_2^{T^+} &= C_A + \frac{7}{3} C_F, & \Gamma_3^{T^+} &= C_A + \frac{23}{6} C_F, \\
\Gamma_3^{T^-} &= \frac{10}{3} C_A + \frac{7}{6} C_F, & \Gamma_2^+ &= 3C_A - \frac{1}{3} C_F = \Gamma_2^-, \\
\Gamma_3^+ &= \begin{pmatrix} \frac{7}{3} C_A + \frac{8}{3} C_F & -\frac{2}{3} C_A + \frac{2}{3} C_F \\ -\frac{4}{3} C_A + \frac{5}{3} C_F & 4C_A + \frac{1}{6} C_F \end{pmatrix}, \\
\Gamma_3^- &= \begin{pmatrix} 4C_A + \frac{1}{6} C_F & -\frac{4}{3} C_A + \frac{5}{3} C_F \\ -\frac{2}{3} C_A + \frac{2}{3} C_F & \frac{7}{3} C_A + \frac{8}{3} C_F \end{pmatrix}. \tag{A.3}
\end{aligned}$$

For massive quarks, the scaling relations receive corrections in $m_s \pm m_q$, depending on the G-parity of the parameter. These corrections are induced by mixing of the operators in (A.1) with twist-2 operators and can be calculated using the light-ray-operator technique of ref. [26] resulting in the following compact expressions:

$$\begin{aligned}
O_3^T(z, vz, 0)^{\mu^2} &= O_3^T(z, vz, 0)^{\mu_0^2} + i \frac{C_F \alpha_s}{2\pi} \ln \frac{\mu_0^2}{\mu^2} \int_0^1 dt \left\{ \frac{m_s}{v} \left[O_2(z, vz) - 2t O_2(z, tvz) \right] \right. \\
&\quad \left. + \frac{m_q}{1-v} \left[O_2(vz, 0) - 2t O_2((1 - (1-v)t)z, 0) \right] \right\} + \dots, \\
O_3(z, vz, 0)^{\mu^2} &= O_3(z, vz, 0)^{\mu_0^2} + i \frac{C_F \alpha_s}{4\pi} \ln \frac{\mu_0^2}{\mu^2} \int_0^1 dt \left\{ \frac{m_s}{v} \left[O_2^T(z, vz) - 2t O_2^T(z, tvz) \right] \right. \\
&\quad \left. + \frac{m_q}{1-v} \left[O_2^T(vz, 0) - 2t O_2^T((1 - (1-v)t)z, 0) \right] \right\} + \dots, \\
\tilde{O}_3(z, vz, 0)^{\mu^2} &= \tilde{O}_3(z, vz, 0)^{\mu_0^2} + i \frac{C_F \alpha_s}{4\pi} \ln \frac{\mu_0^2}{\mu^2} \int_0^1 dt \left\{ -\frac{m_s}{v} \left[O_2^T(z, vz) - 2t O_2^T(z, tvz) \right] \right. \\
&\quad \left. + \frac{m_q}{1-v} \left[O_2^T(vz, 0) - 2t O_2^T((1 - (1-v)t)z, 0) \right] \right\} + \dots \tag{A.4}
\end{aligned}$$

The twist-2 operators in the above relations are given by

$$O_2(az, bz) = \bar{q}(az)\gamma_z s(bz), \quad O_2^T(az, bz) = \bar{q}(az)\sigma_{\perp z} s(bz) \quad (\text{A.5})$$

and the dots stand for $O(\alpha_s)$ contributions from the twist-3 operators.

Taking (A.2) and (A.4) together, one finds that the G-even parameters mix with

$$f_{K^*}^{\perp(\parallel)} \{(m_s + m_q), (m_s - m_q)a_1^{\perp(\parallel)}, (m_s + m_q)a_2^{\perp(\parallel)}\},$$

whereas the G-odd ones mix with

$$f_{K^*}^{\perp(\parallel)} \{(m_s - m_q), (m_s + m_q)a_1^{\perp(\parallel)}, (m_s - m_q)a_2^{\perp(\parallel)}\}.$$

For the \perp parameters, and ζ_3^{\parallel} and κ_3^{\parallel} , the combination of (A.2) and the projection of (A.4) onto the corresponding partial waves results in the scaling relations given in (3.13). For the remaining parameters, one finds the following relations:

$$\begin{aligned} & \begin{pmatrix} f_{K^*}^{\parallel} \omega_{3K^*}^{\parallel} \\ f_{K^*}^{\parallel} \tilde{\omega}_{3K^*}^{\parallel} \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} a_1^{\perp}(K^*) \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} a_2^{\perp}(K^*) \end{pmatrix}^{\mu^2} = L^{\Gamma_{\omega}/\beta_0} \begin{pmatrix} f_{K^*}^{\parallel} \omega_{3K^*}^{\parallel} \\ f_{K^*}^{\parallel} \tilde{\omega}_{3K^*}^{\parallel} \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} a_1^{\perp}(K^*) \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} a_2^{\perp}(K^*) \end{pmatrix}^{\mu_0^2}, \\ & \begin{pmatrix} f_{K^*}^{\parallel} \lambda_{3K^*}^{\parallel} \\ f_{K^*}^{\parallel} \tilde{\lambda}_{3K^*}^{\parallel} \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} a_1^{\perp}(K^*) \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} a_2^{\perp}(K^*) \end{pmatrix}^{\mu^2} = L^{\Gamma_{\lambda}/\beta_0} \begin{pmatrix} f_{K^*}^{\parallel} \lambda_{3K^*}^{\parallel} \\ f_{K^*}^{\parallel} \tilde{\lambda}_{3K^*}^{\parallel} \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} \\ \frac{m_s + m_q}{m_{K^*}} f_{K^*}^{\perp} a_1^{\perp}(K^*) \\ \frac{m_s - m_q}{m_{K^*}} f_{K^*}^{\perp} a_2^{\perp}(K^*) \end{pmatrix}^{\mu_0^2}. \end{aligned} \quad (\text{A.6})$$

The anomalous dimension matrices are given by

$$\Gamma_{\omega} = \begin{pmatrix} \frac{179}{18} & \frac{7}{4} & -\frac{7}{9} & \frac{7}{15} & \frac{2}{3} \\ \frac{1}{3} & \frac{77}{6} & -\frac{2}{45} & -\frac{2}{5} & -\frac{4}{15} \\ 0 & 0 & \frac{16}{3} & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & \frac{88}{9} \end{pmatrix}, \quad \Gamma_{\lambda} = \begin{pmatrix} \frac{77}{6} & \frac{1}{3} & \frac{2}{45} & \frac{2}{5} & \frac{4}{15} \\ \frac{7}{4} & \frac{179}{18} & \frac{7}{9} & -\frac{7}{15} & -\frac{2}{3} \\ 0 & 0 & \frac{16}{3} & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & \frac{88}{9} \end{pmatrix}. \quad (\text{A.7})$$

The resulting explicit expressions for the twist-3 parameters are rather bulky, so we do not give them explicitly.

B. Sum rules for twist-2 matrix elements

In this appendix we list and evaluate the QCD sum rules for twist-2 matrix elements of the K^* . The sum rules for $f_{K^*}^{\parallel,\perp}$, including SU(3)-breaking corrections, were calculated in refs. [17, 29], those for $a_1^{\parallel,\perp}(K^*)$ in ref. [17], and those for $a_2^{\parallel,\perp}(K^*)$ in ref. [15], apart from the perturbative terms in m_s^2 and the radiative corrections to the quark condensate, which are new.

For the longitudinal parameters, the sum rules read:

$$\begin{aligned}
 (f_{K^*}^{\parallel})^2 e^{-m_{K^*}^2/M^2} &= \frac{1}{4\pi^2} \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s-m_s^2)^2(s+2m_s^2)}{s^3} + \frac{\alpha_s}{\pi} \frac{M^2}{4\pi^2} (1 - e^{-s_0/M^2}) \\
 &+ \frac{m_s \langle \bar{s}s \rangle}{M^2} \left(1 + \frac{m_s^2}{3M^2} - \frac{13}{9} \frac{\alpha_s}{\pi} \right) + \frac{4}{3} \frac{\alpha_s}{\pi} \frac{m_s \langle \bar{q}q \rangle}{M^2} + \frac{1}{12M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
 &- \frac{16\pi\alpha_s}{9M^4} \langle \bar{q}q \rangle \langle \bar{s}s \rangle + \frac{16\pi\alpha_s}{81M^4} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2), \tag{B.1}
 \end{aligned}$$

$$\begin{aligned}
 a_1^{\parallel}(K^*) (f_{K^*}^{\parallel})^2 e^{-m_{K^*}^2/M^2} &= \frac{5}{4\pi^2} m_s^4 \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s-m_s^2)^2}{s^4} \\
 &+ \frac{5m_s^2}{18M^4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left(-\frac{1}{2} + \gamma_E - \text{Ei} \left(-\frac{s_0}{M^2} \right) + \ln \frac{m_s^2}{M^2} + \frac{M^2}{s_0} \left(\frac{M^2}{s_0} - 1 \right) e^{-s_0/M^2} \right) \\
 &- \frac{5}{3} \frac{m_s \langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{\alpha_s}{\pi} \left[-\frac{124}{27} + \frac{8}{9} \left(1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei} \left(-\frac{s_0}{M^2} \right) \right) \right] \right\} \\
 &- \frac{5}{3} \frac{m_s^3 \langle \bar{s}s \rangle}{M^4} + \frac{20}{27} \frac{\alpha_s}{\pi} \frac{m_s \langle \bar{q}q \rangle}{M^2} + \frac{5}{9} \frac{m_s \langle \bar{s}\sigma g G s \rangle}{M^4} + \frac{80\pi\alpha_s}{81M^4} (\langle \bar{q}q \rangle^2 - \langle \bar{s}s \rangle^2), \tag{B.2}
 \end{aligned}$$

$$\begin{aligned}
 a_2^{\parallel}(K^*) (f_{K^*}^{\parallel})^2 e^{-m_{K^*}^2/M^2} &= \frac{7}{4\pi^2} m_s^4 \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s-m_s^2)^2(2m_s^2-s)}{s^5} + \tag{B.3} \\
 &+ \frac{7}{72\pi^2} \frac{\alpha_s}{\pi} M^2 (1 - e^{-s_0/M^2}) + \frac{7}{36M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
 &+ \frac{7}{3} \frac{m_s \langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{\alpha_s}{\pi} \left[\frac{184}{27} + \frac{25}{18} \left(1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei} \left(-\frac{s_0}{M^2} \right) \right) \right] \right\} \\
 &+ \frac{49}{27} \frac{\alpha_s}{\pi} \frac{m_s \langle \bar{q}q \rangle}{M^2} - \frac{35}{18} \frac{m_s \langle \bar{s}\sigma g G s \rangle}{M^4} + \frac{224\pi\alpha_s}{81M^4} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2) - \frac{112\pi\alpha_s}{27M^4} \langle \bar{q}q \rangle \langle \bar{s}s \rangle.
 \end{aligned}$$

For the transverse parameters, one has:

$$\begin{aligned}
 (f_{K^*}^{\perp})^2 e^{-m_{K^*}^2/M^2} &= \frac{1}{4\pi^2} \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s-m_s^2)^2(s+2m_s^2)}{s^3} \\
 &+ \frac{1}{4\pi^2} \int_0^{s_0} ds e^{-s/M^2} \frac{\alpha_s}{\pi} \left(\frac{7}{9} + \frac{2}{3} \ln \frac{s}{\mu^2} \right) - \frac{1}{12M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
 &+ \frac{m_s \langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{m_s^2}{3M^2} + \frac{\alpha_s}{\pi} \left(-\frac{22}{9} + \frac{2}{3} \left[1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei} \left(-\frac{s_0}{M^2} \right) \right] \right) \right\} \\
 &- \frac{1}{3M^4} m_s \langle \bar{s}\sigma g G s \rangle - \frac{32\pi\alpha_s}{81M^4} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2), \tag{B.4}
 \end{aligned}$$

$\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3 \text{ GeV}^3$ $\langle \bar{q}\sigma gGq \rangle = m_0^2 \langle \bar{q}q \rangle$ $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = (0.012 \pm 0.003) \text{ GeV}^4$	$\langle \bar{s}s \rangle = (1 - \delta_3) \langle \bar{q}q \rangle$ $\langle \bar{s}\sigma gGs \rangle = (1 - \delta_5) \langle \bar{q}\sigma gGq \rangle$
$m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2, \quad \delta_3 = 0.2 \pm 0.2, \quad \delta_5 = 0.2 \pm 0.2$	
$\bar{m}_s(2 \text{ GeV}) = (100 \pm 20) \text{ MeV} \quad \longleftrightarrow \quad \bar{m}_s(1 \text{ GeV}) = (133 \pm 27) \text{ MeV}$ $\bar{m}_q(\mu) = \bar{m}_s(\mu)/R, \quad R = 24.6 \pm 1.2$	
$\alpha_s(m_Z) = 0.1176 \pm 0.002 \quad \longleftrightarrow \quad \alpha_s(1 \text{ GeV}) = 0.497 \pm 0.005$	

Table 2: Input parameters for sum rules at the renormalisation scale $\mu = 1 \text{ GeV}$. The value of m_s is obtained from unquenched lattice calculations with $n_f = 2$ flavours as summarised in [30], which agrees with the results from QCD sum rule calculations [31]. \bar{m}_q is taken from chiral perturbation theory [32]. $\alpha_s(m_Z)$ is the PDG average [33].

$$\begin{aligned}
 a_1^\perp(K^*)(f_{K^*}^\perp)^2 e^{-m_{K^*}^2/M^2} &= \frac{5}{4\pi^2} m_s^4 \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s - m_s^2)^2}{s^4} + \frac{10}{9} \frac{m_s \langle \bar{s}\sigma gGs \rangle}{M^4} \\
 &+ \frac{5m_s^2}{9M^4} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left(\frac{1}{4} + \gamma_E - \text{Ei} \left(-\frac{s_0}{M^2} \right) + \ln \frac{\mu^2}{M^2} + \frac{M^2}{s_0} \left(\frac{M^2}{s_0} - 1 \right) e^{-s_0/M^2} \right) \\
 &- \frac{5}{3} \frac{m_s \langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{\alpha_s}{\pi} \left[-\frac{49}{9} + \frac{4}{3} \left(1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei} \left(-\frac{s_0}{M^2} \right) \right) \right] \right\}, \quad (\text{B.5})
 \end{aligned}$$

$$\begin{aligned}
 a_2^\perp(K^*)(f_{K^*}^\perp)^2 e^{-m_{K^*}^2/M^2} &= \\
 &\frac{7}{4\pi^2} m_s^4 \int_{m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s - m_s^2)^2 (2m_s^2 - s)}{s^5} + \frac{7}{90\pi^2} \frac{\alpha_s}{\pi} M^2 (1 - e^{-s_0/M^2}) + \frac{7}{54M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
 &+ \frac{7}{3} \frac{m_s \langle \bar{s}s \rangle}{M^2} \left\{ 1 + \frac{\alpha_s}{\pi} \left[-\frac{206}{27} + \frac{16}{9} \left(1 - \gamma_E + \ln \frac{M^2}{\mu^2} + \frac{M^2}{s_0} e^{-s_0/M^2} + \text{Ei} \left(-\frac{s_0}{M^2} \right) \right) \right] \right\} \\
 &- \frac{49}{18} \frac{m_s \langle \bar{s}\sigma gGs \rangle}{M^4} + \frac{112\pi\alpha_s}{81M^4} (\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle^2). \quad (\text{B.6})
 \end{aligned}$$

For ρ and ϕ , one has $a_1^{\parallel,\perp} = 0$. To obtain the sum rules for $f_\phi^{\parallel,\perp}$ and $a_2^{\parallel,\perp}(\phi)$, one has to replace the perturbative contributions to the above sum rules by

$$\begin{aligned}
 \text{for } (f_\phi^{\parallel,\perp})^2: & \quad \frac{1}{4\pi^2} \int_{4m_s^2}^{s_0} ds e^{-s/M^2} \frac{(s + 2m_s^2) \sqrt{1 - 4m_s^2/s}}{s}, \\
 \text{for } a_2^{\parallel,\perp}(\phi)(f_\phi^{\parallel,\perp})^2: & \quad -\frac{7}{2\pi^2} \int_{4m_s^2}^{s_0} ds e^{-s/M^2} \frac{m_s^4 \sqrt{1 - 4m_s^2/s}}{s^2}. \quad (\text{B.7})
 \end{aligned}$$

In addition, one has to substitute $\langle \bar{q}q \rangle \rightarrow \langle \bar{s}s \rangle$ and to double the terms in $m_s \langle \bar{s}s \rangle$, $m_s \langle \bar{q}q \rangle$ and $m_s \langle \bar{s}\sigma gGs \rangle$.

We evaluate the sum rules using the input given in table 2. The results are given in section 2.3 and table 1.

C. Sum rules for twist-3 matrix elements

The chiral-even twist-3 parameters $\zeta_{3K^*}^\parallel, \tilde{\omega}_{3K^*}^\parallel, \tilde{\lambda}_{3K^*}^\parallel$ can be determined from the correlation

function

$$ig_{\alpha\mu}^{\perp} \int d^4y e^{-ipy} \langle 0 | T \bar{q}(z) g \tilde{G}_{\alpha z}(vz) \gamma_z \gamma_5 s(0) \bar{s}(y) \gamma_{\mu} q(y) | 0 \rangle = (pz)^2 (2 - D) \tilde{\Pi}_{3;K^*}^{\parallel}(v, pz); \quad (\text{C.1})$$

D is the number of dimensions. In terms of hadronic contributions, the correlation function is given by

$$\tilde{\Pi}_{3;K^*}^{\parallel}(v, pz) = \frac{(f_{K^*}^{\parallel})^2 m_{K^*}^2}{m_{K^*}^2 - p^2} \int \mathcal{D}(\underline{\alpha}) e^{-ipy(\alpha_2 + v\alpha_3)} \tilde{\Phi}_{3;K^*}^{\parallel}(\underline{\alpha}) + \dots; \quad (\text{C.2})$$

the dots denote contributions from higher-mass states. The parameters $\kappa_{3K^*}^{\parallel}$, $\omega_{3K^*}^{\parallel}$ and $\lambda_{3K^*}^{\parallel}$ can be obtained from an analogous correlation functions $\Pi_{3;K^*}^{\parallel}(v, pz)$ with

$$g \tilde{G}_{\alpha z} \gamma_z \gamma_5 \rightarrow g G_{\alpha z} i \gamma_z.$$

For the chiral-odd operator, one has

$$i \int d^4y e^{-ipy} \langle 0 | T \bar{q}(z) \sigma_{z\mu} g G_{z\mu}(vz) s(0) \bar{s}(y) \sigma_{pz} q(y) | 0 \rangle = i(pz)^3 \Pi_{3;K^*}^{\perp}(v, pz). \quad (\text{C.3})$$

All three correlation functions Π can be written as

$$\Pi_{3;K^*}(v, pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_2 + v\alpha_3)} \pi_{3;K^*}(\underline{\alpha}).$$

For the functions $\pi_{3;K^*}(\underline{\alpha})$ we find

$$\begin{aligned} \pi_{3;K^*}^{\perp}(\underline{\alpha}) &= \frac{\alpha_s}{2\pi^3} \ln \frac{-p^2}{\mu^2} \left[p^2 \alpha_1 \alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_1} \right) \right. \\ &\quad \left. + m_s m_q \frac{\alpha_3^2}{\bar{\alpha}_1 \bar{\alpha}_2} \left[\bar{\alpha}_2 \left(\ln \frac{\alpha_2 \alpha_3}{\bar{\alpha}_1} + \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) - \{\alpha_1 \leftrightarrow \alpha_2\} \right] \right. \\ &\quad \left. + m_s^2 \left\{ -\alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_1} \right) - \frac{\alpha_2 \alpha_3^2}{\bar{\alpha}_2^2} \left(\ln \frac{\alpha_1 \alpha_3}{\bar{\alpha}_2} + \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) \right\} \right. \\ &\quad \left. - m_q^2 \{\alpha_1 \leftrightarrow \alpha_2\} \right] \\ &\quad + \frac{1}{12} \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{\alpha_1 \alpha_2 (\alpha_1 - \alpha_2) \delta(\alpha_3)}{\alpha_1 m_q^2 + \alpha_2 m_s^2 - \alpha_1 \alpha_2 p^2} \\ &\quad + \frac{2}{3p^2} \frac{\alpha_s}{\pi} \left\{ \frac{\bar{\alpha}_3}{2} (1 + \alpha_3) (m_q \langle \bar{q}q \rangle \delta(\alpha_2) - m_s \langle \bar{s}s \rangle \delta(\alpha_1)) \right. \\ &\quad \left. + \alpha_3 \left[1 + \alpha_3 \left(1 + \ln(\alpha_3 \bar{\alpha}_3) + \ln \frac{-p^2}{\mu^2} \right) \right] (m_s \langle \bar{q}q \rangle \delta(\alpha_2) - m_q \langle \bar{s}s \rangle \delta(\alpha_1)) \right\} \\ &\quad + \frac{1}{6p^4} \delta(\alpha_3) \{ m_q \langle \bar{q} \sigma G q \rangle \delta(\alpha_2) - m_s \langle \bar{s} \sigma G s \rangle \delta(\alpha_1) \} \\ &\quad + \frac{16}{27p^4} \pi \alpha_s \delta(\alpha_3) \{ \langle \bar{q}q \rangle^2 \delta(\alpha_2) - \langle \bar{s}s \rangle^2 \delta(\alpha_1) \}, \end{aligned} \quad (\text{C.4})$$

$$\begin{aligned}
 \pi_{3;K^*}^{\parallel}(\underline{\alpha}) &= \frac{\alpha_s}{4\pi^3} \ln \frac{-p^2}{\mu^2} \left[p^2 \alpha_1 \alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_1} \right) \right. \\
 &\quad + m_s m_q \frac{\alpha_3^2}{\bar{\alpha}_1 \bar{\alpha}_2} \left\{ \bar{\alpha}_2 \left(\ln \frac{\alpha_2 \alpha_3}{\bar{\alpha}_1} + \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) - \{ \alpha_1 \leftrightarrow \alpha_2 \} \right\} \\
 &\quad + m_s^2 \left\{ -\alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_1} \right) - \frac{\alpha_2 \alpha_3^2}{\bar{\alpha}_2^2} \left(\ln \frac{\alpha_1 \alpha_3}{\bar{\alpha}_2} + \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) \right\} \\
 &\quad - m_q^2 \{ \alpha_1 \leftrightarrow \alpha_2 \}] \\
 &\quad + \frac{1}{24} \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{\alpha_1 \alpha_2 (\alpha_1 - \alpha_2) \delta(\alpha_3)}{\alpha_2 m_s^2 + \alpha_1 m_q^2 - \alpha_1 \alpha_2 p^2} \\
 &\quad + \frac{1}{3p^2} \frac{\alpha_s}{\pi} \left\{ \frac{\bar{\alpha}_3}{2} (1 + \alpha_3) (m_q \langle \bar{q}q \rangle \delta(\alpha_2) - m_s \langle \bar{s}s \rangle \delta(\alpha_1)) \right. \\
 &\quad + \alpha_3 \left[1 + \alpha_3 \left(\ln(\alpha_3 \bar{\alpha}_3) + \ln \frac{-p^2}{\mu^2} \right) \right] (m_s \langle \bar{q}q \rangle \delta(\alpha_2) - m_q \langle \bar{s}s \rangle \delta(\alpha_1)) \left. \right\} \\
 &\quad + \frac{1}{12p^4} \delta(\alpha_3) \{ m_q \langle \bar{q}\sigma g G q \rangle \delta(\alpha_2) - m_s \langle \bar{s}\sigma g G s \rangle \delta(\alpha_1) \} \\
 &\quad + \frac{8}{27p^4} \alpha_s \pi \delta(\alpha_3) (\langle \bar{q}q \rangle^2 \delta(\alpha_2) - \langle \bar{s}s \rangle^2 \delta(\alpha_1)), \tag{C.5}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\pi}_{3;K^*}^{\parallel}(\underline{\alpha}) &= \frac{\alpha_s}{4\pi^3} \ln \frac{-p^2}{\mu^2} \left[-p^2 \alpha_1 \alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_1} + \frac{1}{\bar{\alpha}_2} \right) \right. \\
 &\quad + m_s m_q \frac{\alpha_3^2}{\bar{\alpha}_1 \bar{\alpha}_2} \left\{ \bar{\alpha}_1 \left(\ln \frac{\alpha_1 \alpha_3}{\bar{\alpha}_2} - \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) + \{ \alpha_1 \leftrightarrow \alpha_2 \} \right\} \\
 &\quad + m_s^2 \left\{ \alpha_2 \alpha_3 \left(\frac{1}{\bar{\alpha}_1} + \frac{1}{\bar{\alpha}_2} \right) + \frac{\alpha_2 \alpha_3^2}{\bar{\alpha}_2^2} \left(\ln \frac{\alpha_1 \alpha_3}{\bar{\alpha}_2} + \frac{1}{2} \ln \frac{-p^2}{\mu^2} \right) \right\} \\
 &\quad + m_q^2 \{ \alpha_1 \leftrightarrow \alpha_2 \}] \\
 &\quad + \frac{1}{24} \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{\alpha_1 \alpha_2 \delta(\alpha_3)}{\alpha_2 m_s^2 + \alpha_1 m_q^2 - \alpha_1 \alpha_2 p^2} \\
 &\quad + \frac{1}{3p^2} \frac{\alpha_s}{\pi} \left\{ \frac{\bar{\alpha}_3^2}{2} (m_s \langle \bar{s}s \rangle \delta(\alpha_1) + m_q \langle \bar{q}q \rangle \delta(\alpha_2)) \right. \\
 &\quad + \alpha_3 \left[1 - \alpha_3 \left(2 + \ln(\alpha_3 \bar{\alpha}_3) + \ln \frac{-p^2}{\mu^2} \right) \right] (m_s \langle \bar{q}q \rangle \delta(\alpha_2) + m_q \langle \bar{s}s \rangle \delta(\alpha_1)) \left. \right\} \\
 &\quad + \frac{1}{12p^4} \delta(\alpha_3) \{ m_q \langle \bar{q}\sigma g G q \rangle \delta(\alpha_2) + m_s \langle \bar{s}\sigma g G s \rangle \delta(\alpha_1) \} \\
 &\quad + \frac{8}{27p^4} \alpha_s \pi \delta(\alpha_3) (\langle \bar{q}q \rangle^2 \delta(\alpha_2) + \langle \bar{s}s \rangle^2 \delta(\alpha_1)) \\
 &\quad + \frac{2}{3p^4} \alpha_s \pi \delta(\alpha_1) \delta(\alpha_2) \langle \bar{q}q \rangle \langle \bar{s}s \rangle. \tag{C.6}
 \end{aligned}$$

Here we have dropped all terms that vanish upon Borelisation. The above expressions include all terms to second order in the quark masses. One comment is in order concerning the contribution of the gluon condensate. Upon integration over α_i , and subsequent expansion in powers of the quark masses, this contribution contains terms in $m_{s,q}^2 \ln(m_{s,q}^2/(-p^2))$, which are long-distance effects and must not appear in the short-distance operator product expansion of the correlation functions (C.1) and (C.3). As discussed in ref. [34], the

appearance of these logarithmic terms is due to the fact that the above expressions are obtained using Wick's theorem to calculate the condensate contributions, which implies that the condensates are normal-ordered: $\langle O \rangle = \langle 0 | : O : | 0 \rangle$. Recasting the operator product expansion in terms of non-normal-ordered operators, all infrared sensitive terms can be absorbed into the corresponding condensates. Indeed, using [34]

$$\langle 0 | \bar{s}gGs | 0 \rangle = \langle 0 | : \bar{s}gGs : | 0 \rangle + \frac{m_s}{2} \ln \frac{m_s^2}{\mu^2} \langle 0 | : \frac{\alpha_s}{\pi} G^2 : | 0 \rangle,$$

and the corresponding formula for q quarks, all terms in $\ln m_{q,s}^2$ can be absorbed into the mixed quark-quark-gluon condensate and the resulting short-distance coefficients can be expanded in powers of $m_{q,s}^2$. In calculating the sum rules, we hence will use

$$\ln \frac{-p^2}{m_{q,s}^2} \rightarrow \ln \frac{-p^2}{\mu^2}.$$

The QCD sum rules for the three hadronic parameters $\kappa_{3K^*}^\perp$, $\omega_{3K^*}^\perp$ and $\lambda_{3K^*}^\perp$ describing the DA $\Phi_{3;K^*}^\perp$, eq. (3.11), to NLO in conformal spin then read:

$$\begin{aligned} \left(f_{K^*}^\perp\right)^2 m_{K^*}^2 e^{-m_{K^*}^2/M^2} \kappa_{3K^*}^\perp &= \int_0^{s_0} e^{-s/M^2} \int \mathcal{D}\underline{\alpha} \frac{1}{\pi} \text{Im}_s \pi_{3;K^*}^\perp(\underline{\alpha}), \\ \left(f_{K^*}^\perp\right)^2 m_{K^*}^2 e^{-m_{K^*}^2/M^2} \frac{1}{14} \omega_{3K^*}^\perp &= \int_0^{s_0} e^{-s/M^2} \int \mathcal{D}\underline{\alpha} (\alpha_1 - \alpha_2) \frac{1}{\pi} \text{Im}_s \pi_{3;K^*}^\perp(\underline{\alpha}), \\ \left(f_{K^*}^\perp\right)^2 m_{K^*}^2 e^{-m_{K^*}^2/M^2} \frac{3}{28} \lambda_{3K^*}^\perp &= \int_0^{s_0} e^{-s/M^2} \int \mathcal{D}\underline{\alpha} \left(\alpha_3 - \frac{3}{7}\right) \frac{1}{\pi} \text{Im}_s \pi_{3;K^*}^\perp(\underline{\alpha}). \end{aligned} \quad (\text{C.7})$$

The formulas for the other parameters are analogous.

We evaluate the above sum rules in the Borel window $M^2 = 1 \text{ GeV}^2$ to 2.5 GeV^2 and using the following values of the continuum threshold s_0 :

$$\begin{aligned} s_0^\parallel(\rho) &= (1.3 \pm 0.3) \text{ GeV}^2, & s_0^\parallel(K^*) &= (1.3 \pm 0.3) \text{ GeV}^2, & s_0^\parallel(\phi) &= (1.4 \pm 0.3) \text{ GeV}^2, \\ s_0^\perp(\rho) &= (1.5 \pm 0.3) \text{ GeV}^2, & s_0^\perp(K^*) &= (1.6 \pm 0.3) \text{ GeV}^2, & s_0^\perp(\phi) &= (1.7 \pm 0.3) \text{ GeV}^2. \end{aligned} \quad (\text{C.8})$$

Numerical results, including the uncertainties from the variation of M^2 , s_0 and the input parameters of table 2, are given in table 1.

D. Loop integrals

For the interested reader, we collect the loop integrals needed for calculating the correlation functions in appendix C.

At one loop, one has (with $z^2 = 0$) [15]:

$$\begin{aligned} \int [d^L k] e^{i f_k k z} \frac{(kz)^n}{(k^2)^a ((k-p)^2)^b} &= (-1)^{a+b} (-p^2)^{D/2-a-b} (pz)^n \frac{\Gamma(a+b-D/2)}{\Gamma(a)\Gamma(b)} \\ &\times \int_0^1 dw e^{i(1-w)f_k pz} w^{D/2-1-b} (1-w)^{D/2+n-1-a}, \end{aligned} \quad (\text{D.1})$$

where the integration measure is defined as $d^D k = i/(4\pi)^2 [d^L k]$ and f_k is an arbitrary numerical factor.

One also needs the following integral:

$$\begin{aligned}
 \int [d^L l] e^{i f_l l z} \frac{(lp)(lz)^j}{(l^2)^c ((l-k)^2)^d} &= (-1)^{\frac{D-4}{2}} (k^2)^{D/2-c-d} (kp)(kz)^j \frac{\Gamma(c+d-D/2)}{\Gamma(c)\Gamma(d)} \times \\
 &\times \int_0^1 du e^{i(1-u)f_l k z} u^{D/2-1-d} (1-u)^{D/2+j-c} \\
 &+ (-1)^{\frac{D-4}{2}} (k^2)^{D/2+1-c-d} (pz)(kz)^{j-1} \frac{\Gamma(c+d-D/2-1)}{2\Gamma(c)\Gamma(d)} \\
 &\times \int_0^1 du e^{i(1-u)f_l k z} u^{D/2-d} (1-u)^{D/2-1+j-c} (j + i f_l (1-u)(kz)) .
 \end{aligned} \tag{D.2}$$

Two-loop integrals are obtained by combining the above one-loop integrals. To obtain the correlation functions as integrals over $\mathcal{D}_{\underline{Q}}$, one has to perform a variable transformation from (u, w) to (α_1, α_2) , where the precise transformation relations are fixed by the ‘‘canonical’’ form of the exponential: $\exp(-ipz\{\alpha_2 + v(1 - \alpha_1 - \alpha_2)\})$. Note that (D.3) contains an extra factor kz in the last line which comes from the Taylor expansion of the exponential. This factor can be made disappear by partial integration of the final result and hence does not appear in the explicit formulas for the correlation functions given in appendix C.

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